HW1

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Problem 1

1. 0
2. No
3. 0

Notes on SPD Matrices, Inner Products, Norms, andMetrics

Problem 1: fact 3: If is SPD, then A is invertible, and is SPD too.

Proof: Let be a SPD matrix. Assume is not invertible, thus there exist vector such that .  
 in contradiction to the fact that is SPD. ()  
Therefore is invertible, meaning exists.

Let be an eigenvalue of , hence:  
According to fact 1, all eigenvalues of are positive. Therefore, all (the eigenvalues of ) are positive, and again from fact 1 we obtain that is SPD.

Problem 2: fact 4: Let be an SPD matrix. Then is an inner product.

Proof: Let Q be an SPD matrix and

We will show that the properties of definition 6 hold.

1. Let , then Let , then from SPD definition,

Problem 3: fact 8: Every norm induces a metric: .

Proof: Let , we will show that the properties of definition 11 hold.

1. Let then, from definition,

Let , then and from definition

Problem 4: fact 10: Let be an matrix and denote its Cholesky decomposition.  
 Then .

Proof: Let be an matrix and denote its Cholesky decomposition.

**Computer Exercise 1:**

a:Shape

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c: A picture containing icon

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e:A picture containing text

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Notes on Convexity

Problem 1:

1. is convex:

Let assume that , so:

This is true because by definition and because we will get that .

1. is convex:

Let assume that , so:

1. is convex:

Let assume that , so:

1. is not convex:

For :

and because , is not convex.

1. is not convex:

For :

and because , is not convex.

Notes on Argmin and Argmax

Problem 1: Let be a function from some set into . Then

Proof: Let be a function from some set into .

Problem 2: For some monotonically non-increasing function, and some it holds that: .

Proof: set as and as .

It is easy to see that .

But now: , while for value , and .

Therefore .

Problem 3: Let be a monotonically non-decreasing function. Let be a function we seek to maximize. Then, .

Proof: By the definition of argmax:

Problem 4: Let depend on . Show that .

Proof: By using Fact 5 for , and , we directly deduce that:

that is because that , and is increasing on [0, ∞].

Problem 5: Let and let depend on .  
Show that .

Proof: From Problem 4 we get that:

From Fact 1 we get that

Now, again from Fact 5, for monotonically increasing function we get that:

And again for monotonically increasing function we get that

And one last time, for monotonically increasing function we get that :

So overall we achieved that:

Notes on Linear Least Squares

Problem 1:

Problem 2:

**Notes on Random Vectors**