HW1

Ilana Pervoi, Pan Eyal

Problem 1

1. 0
2. No
3. 0

Notes on SPD Matrices, Inner Products, Norms, andMetrics

Problem 1: fact 3: If is SPD, then A is invertible, and is SPD too.

Proof: Let be a SPD matrix. Assume is not invertible, thus there exist vector such that .  
 in contradiction to the fact that is SPD. ()  
Therefore is invertible, meaning exists.

Let be an eigenvalue of , hence:  
According to fact 1, all eigenvalues of are positive. Therefore, all (the eigenvalues of ) are positive, and again from fact 1 we obtain that is SPD.

Problem 2: fact 4: Let be an SPD matrix. Then is an inner product.

Proof: Let Q be an SPD matrix and

We will show that the properties of definition 6 hold.

1. Let , then Let , then from SPD definition,

Problem 3: fact 8: Every norm induces a metric: .

Proof: Let , we will show that the properties of definition 11 hold.

1. Let then, from definition,

Let , then and from definition

Problem 4: fact 10: Let be an matrix and denote its Cholesky decomposition.  
 Then .

Proof: Let be an matrix and denote its Cholesky decomposition.

**Computer Exercise 1:**

a:Shape

Description automatically generated b:A picture containing icon

Description automatically generated

c: A picture containing icon

Description automatically generatedd: A picture containing text

Description automatically generated

e:A picture containing text

Description automatically generated